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Research Article

A Simplified Analytical Method for Evaluating Piping Reactions on Pressure Vessel Nozzles

Walther Stikvoort

Independent Consultant, Pressure Equipment Design and Structural Integrity, Assen, Netherlands, Formerly Employed at Kellogg Continental, NAM, NRG, GLT PLUS, and P3 Engineering.

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Abstract - *Nozzle–shell intersections in pressure vessels are critical regions where internal pressure effects interact with external piping reactions, leading to localized stress concentrations. Though there are sophisticated numerical tools and WRC-based methods to assess these effects, they may not be provided regularly to an engineer because of their complexity and inaccessibility. A simplified analytical approach to the evaluation of piping reaction loads on flush, isolated, and radially set-in nozzles of cylindrical and spherical shells with or without reinforcing pads is given in this paper. The proposed method integrates the pressure-area technique, which is generally used in European pressure vessel design codes, and the shrink-ring technique that is grounded on the beams-on-elastic-foundation theory to assess the local stress levels that external forces and moments cause. The Maximum Allowable Working Pressure (MAWP) at the intersection of the nozzle and the shell is then obtained and a pressure utilization factor is defined. This is a ratio of the degree of pressure capacity used by the internal pressure and the foundation of determining an imaginary left over stress to use in adding further piping loads, in accordance with elastic shakedown provisions. Permitted individual axial forces and bending moments are obtained with closed-form expressions and the acceptability of actual piping reactions is established by a linear interaction rule of loads. Flanged nozzles are also subjected to the test of ASME Section VIII, Division 1 flange rating requirements. A worked example is given to show that the method produces conservative and physically consistent results, which makes the effects of nozzle reinforcement efficiency and geometry on permissible external loads visible. The technique offers a clear, code-conformable and viable alternative to preliminary design and integrity study of pressure vessel nozzles subjected to combined pressure and piping stresses, but does not exclude the possibility that more detailed local stress or fatigue analyses may still be required for critical applications.*

Keywords - *Pressure vessel nozzles, Nozzle–shell intersection, Piping reactions, Pressure–area method, Shrink-ring method, Beam on elastic foundation, Maximum allowable working pressure (MAWP), Pressure utilization factor, Reinforcing pads, Linear load interaction, ASME Section VIII, Elastic shakedown.*

I. INTRODUCTION

Nozzle–shell intersections in pressure vessels are localized regions of elevated stress concentration, where internal pressure effects interact with external mechanical loads transmitted by attached piping systems. Besides the pressure-induced membrane and bending stresses, nozzles have axial forces, bending moments, and torsional loads that are referred to as piping reactions due to piping weight, thermal contraction or expansion, and system restraints. Such combined effects should be assessed holistically in order to achieve structural integrity and code compliance. Even though there are a number of numerical tools available to assess piping responses on nozzles like FE/Pipe, NozzlePro, and PASS-FEM design engineers are not always equipped with such tools or familiar with their usage. To address this gap, a simplified analytical method has been developed to assist in the evaluation of piping reactions on pressure vessel nozzles.

The proposed method provides an alternative to approaches prescribed in various pressure vessel codes, including AD 2000 [1], EN 13445 [2], PD 5500 [3], and CODAP [4], which are based on analytical limit-load concepts. It also offers an alternative to the methods presented in WRC Bulletins 107 [6], 297 [7], and 368 [8].

While the WRC methods yield more refined stress estimates, they are computationally intensive and require explicit inclusion of pressure-induced stresses as input. The developed simplified method applies to flush, isolated, and set-in nozzles in cylindrical and spherical shells, with or without reinforcing pads. Shear stresses arising from transverse forces and torsional moments at the nozzle–shell interface are neglected, as they typically do not exceed 0.15 times the relevant design stress and therefore have a negligible influence on the overall stress state.

II. BASIS OF THE DEVELOPED SIMPLIFIED EVALUATION METHOD

The technique is a composite of the pressure-area technique used to calculate the pressure capacity at the nozzle-shell intersection with the shrink-ring technique initially formulated by the M.W. Kellogg Company used to assess local stresses caused by piping responses. Shrink-ring method is held on the beams-on-elastic-foundation theory. The present formulation builds on studies referenced in [9] and [10] where the author had a close interest. The Pressure-Area approach, which is used in several European design codes, is used to calculate the Maximum Allowable Working Pressure (MAWP) at the nozzle-shell interface. In this method, the effective lengths of the nozzle neck and shell are factored in by using k-factors that have an effect on the pressure-loaded area and also load-carrying cross-sectional area.

The pressure utilization factor, γ, is defined as:

$$\gamma = \frac{p_d}{\text{MAWP}_{\text{intersection}}}$$

Values of γ less than 1.0 indicate an overdesigned nozzle (i.e., excess reinforcement). Lower γ values correspond to a greater available stress margin for external loads. Accordingly, the fictitious remaining stress available for additional loading is given by:

$$\sigma = (3 - 2\gamma) f_s$$

which satisfies the elastic shakedown criterion.

This criterion is used to restrict the sum of the primary (pressure) and secondary (local bending due to external loads) stresses to assure shakedown to elastic action to prevent progressive plastic deformation. In nozzles with sufficient reinforcement, the stress intensity because of internal pressure should not exceed $2f_s$. Incorporating the pressure utilization factor γ yields the available stress for external loads as expressed above.

The relations for calculating the individually allowable loads are derived from references [9] and [10]. When piping reactions are known, or can be obtained from a nozzle load standard applicable to the pressure vessel, a linear load interaction rule may be applied. This rule implies adding the ratios between the actual loads and their allowable values. Finally, the nozzle flange rating is checked as per the UG-44(b)(4) of [5], either by calculating the allowable axial force F_E and resulting bending moment M_E, or by checking the rating against known applied loads.

III. SUMMARY OF FORMULAS SUPPORTING THE PRESSURE–AREA METHODOLOGY

Table 1. Closed-Form Pressure–Area Relationships for Determining MAWP at Nozzle–Shell Junctions

FLUSH ISOLATED SET-IN NOZZLE WITHOUT REINFORCING PAD IN CYLINDRICAL SHELL	FLUSH ISOLATED SET-IN NOZZLE WITH REINFORCING PAD IN CYLINDRICAL SHELL
$L_n = k_n\sqrt{(d_n - t_n)t_n}$	
$L_s = k_s\sqrt{(D_o - t_s)t_s}$	
$A_p = \frac{D_i}{2}\left(L_s + \frac{d_n}{2}\right) + \frac{d_i}{2}(L_n + t_s)$	
$A_f = t_s \cdot L_s + (L_n + t_s)t_n \cdot \left(\frac{f_n}{f_s}\right)$	$A_f = t_s \cdot L_s + (L_n + t_s)t_n \cdot \left(\frac{f_n}{f_s}\right) + k_p\left(L_p \cdot t_p\right)\left(\frac{f_p}{f_s}\right)$

<table>
<tr><td>$\left(\frac{f_n}{f_s}\right)_{\max} = 1.0$</td><td>$\left(\frac{f_n}{f_s}\right)_{\max} = 1.0, \quad \left(\frac{f_p}{f_s}\right)_{\max} = 1.0$</td></tr>
<tr><td colspan="2">$\text{MAWP}_{\text{intersection}} = \frac{f_s}{\left(\frac{A_p}{A_f} + 0.5\right)}$</td></tr>
<tr><td colspan="2">$\text{MAWP}_{\text{undisturbed shell}} = f_s \cdot \ln\left(\frac{D_o}{D_i}\right)$
$\text{MAWP}_{\text{intersection}} \geq \text{MAWP}_{\text{undisturbed shell}}$</td></tr>
<tr><td>FLUSH ISOLATED SET-IN NOZZLE WITHOUT REINFORCING PAD IN SPHERICAL SHELL</td><td>FLUSH ISOLATED SET-IN NOZZLE WITH REINFORCING PAD IN SPHERICAL SHELL</td></tr>
<tr><td colspan="2">$L_n = k_n\sqrt{(d_n - t_n)t_n}$</td></tr>
<tr><td colspan="2">$L_s = k_s\sqrt{(2R_i + t_s)t_s}$</td></tr>
<tr><td colspan="2">$R_m = R_i + 0.5t_s$</td></tr>
<tr><td colspan="2">$\delta = \frac{d_n}{2R_m}$</td></tr>
<tr><td colspan="2">$a = R_m \cdot \arcsin\delta$</td></tr>
<tr><td colspan="2">$A_p = 0.5R_i^2 \frac{L_s + a}{0.5t_s + R_i} + 0.5d_i(L_n + t_s)$</td></tr>
<tr><td>$A_f = t_s \cdot L_s + (L_n + t_s)t_n \cdot \left(\frac{f_n}{f_s}\right)$
$\left(\frac{f_n}{f_s}\right)_{\max} = 1.0$</td><td>$A_f = t_s \cdot L_s + (L_n + t_s)t_n \cdot \left(\frac{f_n}{f_s}\right) + k_p(L_p \cdot t_p)\left(\frac{f_p}{f_s}\right)$
$\left(\frac{f_n}{f_s}\right)_{\max} = 1.0, \quad \left(\frac{f_p}{f_s}\right)_{\max} = 1.0$</td></tr>
<tr><td colspan="2">$\text{MAWP}_{\text{intersection}} = \frac{f_s}{\left(\frac{A_p}{A_f} + 0.5\right)}$</td></tr>
<tr><td colspan="2">$\text{MAWP}_{\text{undisturbed shell}} = 2f_s \cdot \ln\left(\frac{D_o}{D_i}\right)$</td></tr>
</table>

From closer examination of the expression for the MAWP at a nozzle-shell intersection in a pressure vessel:

$$\text{MAWP}_{\text{intersection}} = \frac{f_s}{\left(\frac{A_p}{A_f} + 0.5\right)}$$

it is mathematically evident that as the ratio $\frac{A_p}{A_f}$ decreases, the MAWP increases. This inverse relationship highlights the importance of maximizing the reinforcement area A_f relative to the projected pressure area A_p compared to enhance the pressure-holding capacity at the nozzle intersection.

A. Overview of k-Factors

Table 2. Definition and Typical Ranges of k-Factors Used in the Pressure–Area Method

k-factor	Description	Typical Range
k_s	Shell factor	0.75 – 1.0
k_n	Nozzle factor	0.78 – 1.25
k_p	Reinforcing pad (repad) factor	0.75 – 1.0

The efficiency factors k_s and k_n are determined based on the dimensionless geometric parameter λ, defined as:

$$\lambda = \frac{d_i}{\sqrt{(D_o - t_s)t_s}}$$

The efficiency factor k_n remains constant for all values of λ and is given by:

$$k_n = 1.0$$

In contrast, the efficiency factor k_s varies as a function of λ, according to the following piecewise definition:

$$k_s = \begin{cases} 1, & \text{for } \lambda \leq 4 \\ \frac{13}{12} - \frac{\lambda}{48}, & \text{for } 4 < \lambda < 16 \\ 0.75, & \text{for } \lambda \geq 16 \end{cases}$$

Linear interpolation is permitted for values of λ between 4 and 16 to ensure a smooth transition.

The designer is allowed to choose k-values but not their choice is determined by the governing design code or standard.

Table 3. Nomenclature

Symbol	Description	Unit
D_o	Outside diameter shell	mm
D_i	Inside diameter shell	mm
R_i	Inside radius sphere	mm
R_m	Mean radius sphere	mm
t_s	Shell thickness	mm
t_n	Nozzle neck thickness	mm
d_n	Outside nozzle diameter	mm
d_i	Inside nozzle diameter	mm
L_p	Width of reinforcing pad	mm
d_p	Outside diameter repad ($2L_p + d_n$)	mm
t_p	Thickness of reinforcing pad	mm
L_n	Effective length of nozzle neck	mm
L_s	Effective length of shell	mm
k_s	Shell factor	–
k_n	Nozzle factor	–
k_p	Reinforcing pad (repad) factor	–
A_p	Pressure loaded area	mm^2
A_f	Load-carrying cross-sectional area	mm^2
p_d	Internal design pressure	MPa
$MAWP_{intersection}$	Maximum Allowable Working Pressure for nozzle intersection	MPa
$MAWP_{undisturbed\ shell}$	Maximum Allowable Working Pressure for undisturbed shell	MPa
σ	Remaining stress available for additional loads	MPa
f_s	Design stress shell	MPa

Symbol	Description	Unit
f_n	Design stress nozzle	MPa
f_p	Design stress reinforcing pad	MPa
ln	Natural logarithm	–

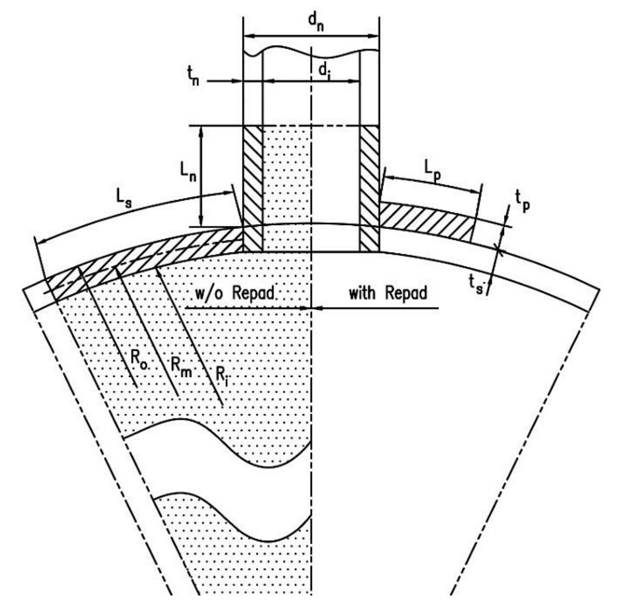

Figure 1. Nozzle Configuration on Spherical Shell

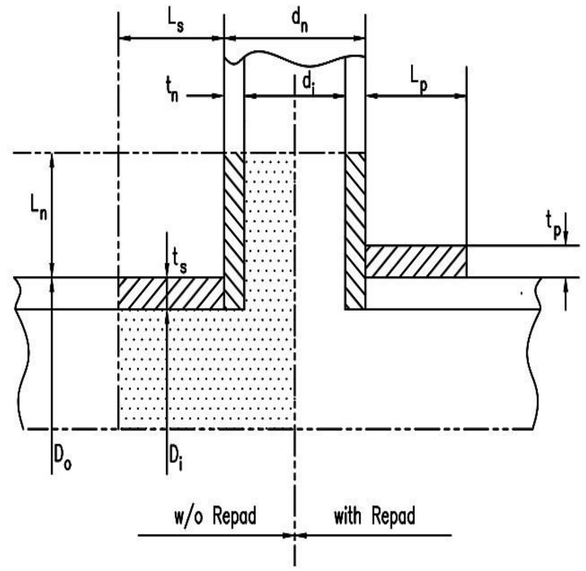

Figure 2. Nozzle Configuration on Cylindrical Shell

B. Utilization Factor

The pressure utilization factor is defined as:

$$\gamma = \frac{p_d}{\mathrm{MAWP}_{\mathrm{intersection}}}$$

C. Remaining Strength Available for Additional Loadings (Piping Reactions)

The fictitious remaining strength (stress) to be used in further loadings is calculated as:

$\sigma = (3 - 2\gamma) f_s$ which satisfies the elastic shakedown criterion.

Table 4. Formulas for Individual Allowable Loads and Auxiliary Coefficients for Nozzles on Cylindrical and Spherical Shells

Calculation of Individual Allowable Loads	
Nozzles on Cylindrical Shell	**Nozzles on Spherical Shell**
$F = \frac{\sigma}{6\, c_{21}}$	$F = \frac{\sigma}{1.75\, c_{21}}$
$M_l = \frac{\sigma}{1.5\, c_{31}}$	$M = \frac{\sigma}{1.75\, c_{31}}$
$M_c = \frac{\sigma}{1.15\, c_{31}\, c_{41}}$	
Auxiliary Values for Both Cases	
$c_{11} = \frac{D_o - t_s}{2 t_s}$	

$c_{21}=\dfrac{\sqrt{c_{11}}}{\pi t_s d_n}$	
$c_{31}=\dfrac{4\sqrt{c_{11}}}{\pi t_s d_n^2}$	
$c_{41}=\sqrt{\dfrac{d_n}{2t_s}}$	
$c_n=\dfrac{t_s}{t_n}$	
With Reinforcing Pad Present Replace t_s with t_s+t_p in the formulas for c_{11}, c_{21}, c_{31}, c_{41}, and c_n. **Transition Between Vessel Shell and Reinforcing Pad**	
Cylindrical shell	**Spherical shell**
$F=\dfrac{\sigma}{6\,c_{22}}$	$F=\dfrac{\sigma}{1.75\,c_{22}}$
$M_l=\dfrac{\sigma}{1.5\,c_{32}}$	$M=\dfrac{\sigma}{1.75\,c_{32}}$
$M_c=\dfrac{\sigma}{1.15\,c_{32}\,.\,c_{42}}$	
For the auxiliary values c_{12}, c_{22}, c_{32} and c_{42} in both cases:	
$c_{12}=\dfrac{D_{o}-t_s}{2t_s}$	
$c_{22}=\dfrac{\sqrt{c_{12}}}{\pi.t_s(d_p)}$	
$c_{32}=\dfrac{4\sqrt{c_{12}}}{\pi\,t_s(d_p)^2}$	
$c_{42}=\sqrt{\dfrac{d_p}{2t_s}}$	

IV. APPLICATION OF THE LINEAR LOAD INTERACTION RULE

Once the Maximum Allowable Working Pressure at the intersection of the nozzle ($MAWP_{intersection}$) and the pressure utilization factor (γ) have been determined, the fictitious remaining stress (σ) can be calculated. With the aid of the formulas of the auxiliary values obtained in the foregoing section, this residual stress is applied to obtain the individually allowable loads (axial force F_{allow}, and bending moments $M_{l,\text{allow}}$ and $M_{c,\text{allow}}$).

However, piping systems normally impose a mixture of these loads at the same time. A linear interaction rule of load is used to test the acceptability of the actual combined loads (F, M_l, M_c) obtained from a piping stress analysis, a linear load interaction rule is applied. This rule sums the ratios of the applied loads to their allowable counterparts. The actual piping reactions are considered acceptable if the resulting interaction value is equal to or less than 1.0:

$$\frac{F}{F_{\text{allow}}}+\frac{M_l}{M_{l,\text{allow}}}+\frac{M_c}{M_{c,\text{allow}}}\leq 1.0$$

where:

- F = applied axial force on the nozzle
- F_{allow} = allowable axial force on nozzle
- M_l = applied longitudinal moment on the nozzle

- $M_{l,\text{allow}}$ = allowable longitudinal moment on the nozzle
- M_c = applied circumferential moment on the nozzle
- $M_{c,\text{allow}}$ = allowable circumferential moment on the nozzle

V. FLANGE RATING VERIFICATION

The preceding sections assess the structural sufficiency of the intersection of the nozzle and shell when subjected to combined pressure and piping loads. In nozzles that have a flange, a further check is necessary to be sure that the flange is not overloaded. This verification is separate to the shell assessment and is conducted in compliance with ASME, Section VIII, Division 1, paragraph UG-44(b)(4) [5], as discussed below.

This verification ensures that the external piping loads, when added to the internal pressure do not surpass the pressure-temperature rating of the standard flange.

A. Governing Expression

$$16M_E + 4F_E G \leq \pi G^3[(P_R - P_D) + F_M P_R]$$

B. Allowable Individual External Loads

- **Allowable External Tensile Axial Force,** F_E (for $M_E = 0$):

$$F_E = \frac{\pi G^2}{4}[(P_R - P_D) + F_M P_R]$$

- **Allowable External Bending Moment,** M_E (for $F_E = 0$):

$$M_E = \frac{\pi G^3}{16}[(P_R - P_D) + F_M P_R]$$

C. Remarks

- The equations are applicable in individual flange loading as per ASME Section VIII, Division 1.
- Any symbols and nomenclature are specified according to ASME IX-VIII-1, Paragraph UG-44(b) (4).

D. Remarks

For an NPS 12" (DN 300), Class 300, Weld Neck flange, to ASME B16.5, made of A105 material, and with a spiral wound gasket to ASME B16.20, the following data apply:

Rated pressure: 45.1 bar @ 150°C = 4.51 MPa

Design pressure = 2.5 MPa

G = effective gasket diameter = 357.5 mm

F_M = 0.5

$$F_E = \frac{\pi \times 357.5^2}{4}[(4.51 - 2.5) + 0.5 \times 4.51] = 428115\ \text{N}$$

$$M_E = \frac{\pi \times 357.5^3}{16}[(4.51 - 2.5) + 0.5 \times 4.51] = 38262828\ \text{N·mm}$$

E. Alternative Methodology: Check of Flange Rating for Known External Piping Reaction Loads on Nozzle

$$F_E = 16200\ \text{N}$$

$$M_l = 12200000\ \text{N·mm}$$

$$M_c = 8300000\ \text{N·mm}$$

$$M_E = \sqrt{12200000^2 + 8300000^2} = 14755677 \text{ N·mm}$$

F. Governing Check

$$\frac{\left(\frac{16M_E + 4F_E G}{\pi G^3}\right) + P_D}{1 + F_M} \leq P_R$$

$$\frac{\left(\frac{16 \times 14755677 + 4 \times 16200 \times 357.5}{\pi \times 357.5^3}\right) + 2.5}{1 + 0.5} = 2.87 \text{ MPa} < 4.51 \text{ MPa}$$

Conclusion: The flange satisfies the requirements of ASME Section VIII, Division 1, Paragraph UG-44(b)(4).

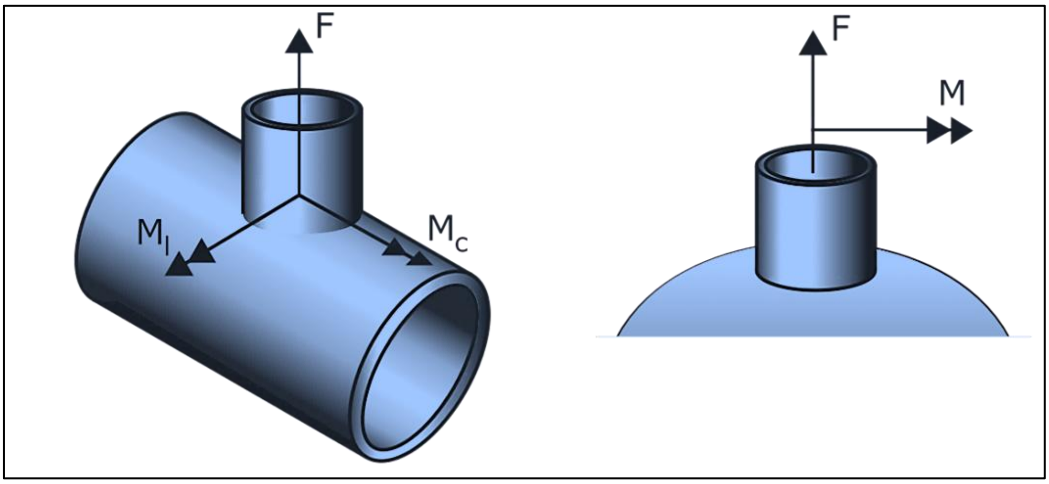

Figure 3. Conventions for Applied Forces and Moments Acting on a Nozzle

VI. WORKED EXAMPLE OF A RADIAL ISOLATED FLUSH SET-IN NOZZLE IN A CYLINDRICAL SHELL WITH REINFORCING PAD

Table 5. Vessel Input Data

Symbol	Description	Value	Unit
D_o	Outside diameter shell	1200	mm
D_i	Inside diameter shell	1171.6	mm
t_s	Shell thickness	14.2	mm
t_n	Nozzle neck thickness	9.6125	mm
d_n	Outside nozzle diameter	323.8	mm
d_i	Inside nozzle diameter	304.575	mm
L_p	Width of reinforcing pad	80	mm
d_p	Outside diameter repad $(2L_p + d_n)$	483.8	mm
t_p	Thickness of reinforcing pad	15.7	mm
k_s	Shell factor	1.0	–
k_n	Nozzle factor	1.0	–
k_p	Reinforcing pad factor	0.75	–
p_d	Internal design pressure	2.5	MPa
f_s	Design stress shell	154.667	MPa
f_n	Design stress nozzle	142.667	MPa
f_p	Design stress reinforcing pad	154.667	MPa
S_y	Yield strength shell material	232	MPa

A. Calculation

$$\lambda = \frac{d_i}{\sqrt{(D_o - t_s)t_s}} = \frac{304.575}{\sqrt{(1200 - 14.2) \times 14.2}} = 2.3472$$

$$L_n = k_n\sqrt{(d_n - t_n)t_n} = 1.0\sqrt{(323.8 - 9.6125) \times 9.6125} = 54.956\,\text{mm}$$

$$L_s = k_s\sqrt{(D_o - t_s)t_s} = 1.0\sqrt{(1200 - 14.2) \times 14.2} = 129.763\,\text{mm}$$

$$A_p = \frac{D_i}{2}\left(L_s + \frac{d_n}{2}\right) + \frac{d_i}{2}(L_n + t_s) = 182116.937\,\text{mm}^2$$

$$A_f = t_s \cdot L_s + (L_n + t_s)t_n \cdot \left(\frac{f_n}{f_s}\right) + k_p(L_p \cdot t_p)\left(\frac{f_p}{f_s}\right) = 3397.82\,\text{mm}^2$$

$$\text{MAWP}_{\text{intersection}} = \frac{f_s}{\left(\frac{A_p}{A_f} + 0.5\right)} = 2.859\,\text{MPa} > p_d = 2.5\,\text{MPa}$$

$$\text{MAWP}_{\text{undisturbed shell}} = f_s \cdot \ln\left(\frac{D_o}{D_i}\right) = 3.704\,\text{MPa} > p_d = 2.5\,\text{MPa}$$

$$\gamma = \frac{p_d}{\text{MAWP}_{\text{intersection}}} = \frac{2.5}{2.859} = 0.87443$$

$$\sigma = (3 - 2\gamma)f_s = (3 - 2 \times 0.87443) \times 154.667 = 193.514\,\text{MPa}$$

B. Auxiliary Quantities

$$c_{11} = \frac{D_o - (t_s + t_p)}{2(t_s + t_p)} = \frac{1200 - 29.9}{2 \times 29.9} = 19.5669$$

$$c_{21} = \frac{\sqrt{c_{11}}}{\pi(t_s + t_p)d_n} = \frac{\sqrt{19.5669}}{\pi \times 29.9 \times 323.8} = 0.0001454$$

$$c_{31} = \frac{4\sqrt{c_{11}}}{\pi(t_s + t_p)d_n^2} = \frac{4\sqrt{19.5669}}{\pi \times 29.9 \times (323.8)^2} = 0.000001797$$

$$c_{41} = \sqrt{\frac{d_n}{2(t_s + t_p)}} = \sqrt{\frac{323.8}{2 \times 29.9}} = 2.32695$$

$$c_n = \frac{t_s + t_p}{t_n} = \frac{29.9}{9.6125} = 3.1105$$

$$c_{12} = \frac{D_o - t_s}{2t_s} = \frac{1200 - 14.2}{2 \times 14.2} = 41.754$$

$$c_{22} = \frac{\sqrt{c_{12}}}{\pi t_s d_p} = \frac{\sqrt{41.754}}{\pi \times 14.2 \times 483.8} = 0.000299395$$

$$c_{32} = \frac{4\sqrt{c_{12}}}{\pi t_s d_p^2} = \frac{4\sqrt{41.754}}{\pi \times 14.2 \times (483.8)^2} = 0.000002475$$

$$c_{42} = \sqrt{\frac{d_p}{2t_s}} = \sqrt{\frac{483.8}{2 \times 14.2}} = 4.12737$$

C. Allowable Loads

Table 6. Calculated Allowable Axial Force and Bending Moments at Critical Locations of the Nozzle–Shell Junction

Adjacent to nozzle neck	Transition repad - shell
$\mathrm{F} = \frac{\sigma}{6\ c_{21}} = \frac{193.514}{6 \times 0.0001454} = 221818\ \mathrm{N}$	$\mathrm{F} = \frac{\sigma}{6\ c_{22\ F}} = \frac{193.514}{6 \times 0.000299395} = 107725\ \mathrm{N}$
$\mathrm{M_l} = \frac{\sigma}{1.5\ c_{31}} = \frac{193.514}{1.5 \times\ 0.000001797} =$ 71791504 Nmm	$\mathrm{M_l} = \frac{\sigma}{1.5\ c_{32}} = \frac{193.514}{1.5 \times 0.000002475} = 52124983\ \mathrm{Nmm}$
$\mathrm{M_c} = \frac{\sigma}{1.15\ c_{31} . c_{41}} = \frac{193.514}{1.15 \times\ 0.000001797 \times 2.32695} =$ 40241987 Nmm	$\mathrm{M_c} = \frac{\sigma}{1.15\ c_{32} . c_{42}} = \frac{193.514}{1.15 \times\ 0.000002475 \times 4.12737} = 16472744$ Nmm

D. Evaluation of Actual Piping Reactions on ND 300 Nozzle

$$F = 16200\ \mathrm{N}$$
$$M_l = 12200000\ \mathrm{N{\cdot}mm}$$
$$M_c = 8300000\ \mathrm{N{\cdot}mm}$$

E. Linear Load Interaction Rule (LLIR)

a. Adjacent to Nozzle Neck

$$\frac{16200}{221818} + \frac{12200000}{71791504} + \frac{8300000}{40241987} = 0.44922 < 1.0$$

b. Transition Repad-Shell

$$\frac{16200}{107725} + \frac{12200000}{52124983} + \frac{8300000}{16472744} = 0.88826 < 1.0$$

VII. CONCLUSIONS

The paper has provided a simplified, clear analytical approach to assessing piping reaction loads acting upon flush, isolated, radially set-in nozzles in cylindrical and spherical shells of pressure vessels, with or without reinforcing pads. The technique is a viable substitute to more complex numerical procedures and WRC-based methods, but is compatible with the design philosophy of traditional pressure vessel codes, including AD 2000[1], EN 13445[2], PD 5500[3], CODAP[4], and ASME Section VIII[5]. The essence of the proposed method is the calculation of the Maximum Allowable Working Pressure (MAWP) at the intersection point of the nozzle and shell by the pressure-area method. Based on this a utilization factor, γ, has been defined, which gives a direct and quantitative measure of the extent to which the internal pressure uses up the local pressure capacity. By comparing this factor of utilization to a stress criterion based on the stress-strain curve during shakedown, an imaginary residual stress is determined, or the remaining strength with which other external loads can be added to the piping system being connected.

Closed-form expressions are used to convert this remaining stress into individually allowable axial forces and bending moments expressed as closed-form terms, and evaluated by a linear load interaction rule. The formulation enables the reaction of combined piping to be easily and physically relevant. In the case of nozzles that have flanges, the structural sufficiency of the intersection between the nozzle and shell intersection is complemented by an independent flange rating verification in accordance with ASME Section VIII, Division 1, UG-44(b)(5).

The worked example demonstrates that the method produces conservative and consistent results. The values of interaction are below unity in both nozzle neck region and the shell-reinforcing pad transition which verifies the acceptability of the applied piping loads. It is also interesting to observe that the sensitivity of permissible external loads to nozzle geometry, reinforcement efficiency and material properties is clearly shown in the example as evidenced by the significance of balanced nozzle design.

A key insight provided by the method is the explicit role of nozzle overdesign, as reflected by the pressure utilization factor γ. Large values of γ. confirm that the pressure capacity of the intersection at the nozzle is heavily absorbed in internal pressure and therefore there is little available to piping reactions. On the other hand, lower g gives higher values of excess reinforcement and the allowable external loads are higher. This association provides rational grounds on which designers and integrity engineers can determine the trade-off between pressure design efficiency and external load capacity.

In summary, the proposed method:

- Provides a code-consistent, tool to preliminary design and integrity-assess pressure vessel nozzles exposed to combined pressure and piping loads
- Offers clear physical insight into the interaction between pressure utilization, reinforcement effectiveness, and permissible outside reactions;
- Generates conservative results that can be used to vessels that are mainly subjected to the static loading conditions.

While the method is not intended to replace detailed finite element analyses or fatigue tests in applications with severe cyclic loading and complicated geometries or critical consequence of failure, and the method is a robust and effective approach when performing routine engineering checking and compliance testing of pressure vessel nozzles. The methodology as shown in this paper is very appropriate in the spreadsheet implementation, facilitating accurate and reproducible calculations for design engineering tasks.

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